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Dossier Pierre Duhem

Duhem's Critical Analysis of Mechanicism and his Defense of a Formal Conception of Theoretical Physics

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Abstract:
The aim of this paper is to present Duhem's critical view of the dynamical development of mechanics according to two principles of his theory of the development of physics: the continuous and the rational development of physics. These two principles impose a formal conception of physics that aims at demarcating physics from the metaphysical view on the one hand and the pragmatist/conventionalist view on the other hand. Duhem pursues an intermediary conception of physics, a representational system of empirical laws based upon formal principles. This formal conception of physics will adjust to his idea of scientific progress in the form of a sequence of representational systems as structures of increasing comprehensiveness of empirical laws, which leads him to defend a convergent structural realism pointing to an ideal physical theory.

Keywords:
Pierre Duhem; mechanicism; theoretical physics; synthetical / analytical method; structural realism

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Introduction

The aim of this paper is to present Duhem's critical view of the development of mechanics according to two principles of his dynamical theory of the development of physics: the continuous and the rational development of physics (Duhem 1980, 188). These two principles impose a formal conception of physics that aims at demarcating physics from the metaphysical view which searches for causal explanation of physical phenomena, on the one hand and the pragmatist/conventionalist view, with its defense of the principle of undertermination of theories by data on the other hand (Chiappin 1989, 131, 93; Duhem 91, 330; Poincaré 1901, vi). Duhem pursues a formal conception of physics that he defines as a representational system of empirical laws based upon formal principles (Duhem 1974, 19; 1902b, 5), a middle way between

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these two conceptions (Chiappin 1989, iv, 92-93, 121, 243). He constructs this formal conception in such a way that he ends up with an idea of scientific progress in the form of a sequence of representational systems as structures of increasing comprehensiveness of empirical laws, which leads him to defend a convergent structural realism toward an ideal physical theory (Chiappin 1989, 198, 110-116, 198-210).

Duhem rejects a conception of physics that searches for causal explanation, which he deems metaphysical and whose origins, as he points out, lie in the emergent mechanicism of Descartes's rationalistic program. For Duhem, mechanicism – understood as a kind of large-scale mechanism – was a strategy of scientists such as Descartes and Galileo to mathematize nature. Despite rejecting the mechanistic approach to physics, Duhem values the framework of the emergent rationalist program that contains it, together with its demand for the principle of unity and the formal or mathematical organization of science (Duhem 1892, 170).

However, his critical analysis of the mechanistic point of view of physics is more complex than it seems at first sight, since it envisages the dynamical development of mechanics according to two approaches, the synthetic and the analytical. Each one of them has its own vices and virtues, according to Duhem's principles of the continuous and rational development of physics (Duhem 1974, 270, 296; 1980, 188, Chiappin 1989, 77, 80, 91). Both principles are the coordinating principles of a proposal to construct a dynamical conception of physics made of a sequence of more encompassing theories that move systematically toward an ideal conception of physics which pretends to mirror the structural relations between the empirical laws (Duhem 1893, 298, 368-369; 1917, 157; Chiappin 1989, 198, 106-113, 243).

Duhem's Description of Mechanical Theory and the Mechanistic Program

Duhem's critical analysis of the evolution of mechanics and of the conception of physics associated with it is, obviously, made from a historical point of view. Besides allowing him to find a possible tendency in the development of physics, it also enables him to construct a tradition and a historical support for his own formal conception of physics.

In his historical-critical analysis of mechanicism (Chiappin 1989, 18-19), Duhem provides the following general description of the nature of mechanical theory:

> Let us seek to account exactly for the nature of what one calls a mechanical theory. In a mechanical theory one imposes [besides symbolization] on all physical magnitudes on which rely the laws that one has to tie with one another the condition that they be composed by means of the geometrical and mechanical elements of a certain fictitious system; one imposes on all hypotheses that they be the propositions [*énoncés*] of the dynamical properties of this system (Duhem 1892, 154).

This means in mechanical theory that: *i)* the physical concepts used in the empirical laws must be defined in terms of geometrical and mechanical properties of a material system (for example, particles in motion in Descartes's view, matter and repulsive/attractive forces in the mechanical-molecular tradition of Laplace and Poisson, the continuous medium of Fresnel's and Maxwell's ether); *ii)* there is an additional supposition that these mechanical concepts are restricted to mass, size, motion and/or force. Duhem spells out this additional condition as follows: "When we propose to construct a mechanical theory, we impose upon ourselves another obligation which consists in putting, into these definitions and hypotheses, only a very restricted number of notions of a determined nature [mass, size, motion and/or force]" (Duhem 1892, 156).

These two conditions define the core of the physical content of mechanical theory. The basic mechanical concepts of mass, motion, size and/or force are mechanical properties of a mechanical system. The kinetic, or Cartesian view, excludes force from the definition of mechanical explanation. It views change of motion as the result of collisions. The dynamical or Newtonian view of mechanical theory includes force as a primitive concept. These constitute two general and competing programs to explain physical phenomena mechanically.

The mechanistic program is characterized by additional principles: *iii)* commitment to a set of propositions (in the form of equations) by means of which the general features of physical phenomena are described (these propositions are the fundamental laws of mechanics); *iv)* commitment to the application of the principle of logical unity in physics (Duhem 1974, 91; Chiappin 1989, 178, 240-241), which requires a single theoretical system to account for physical phenomena; *v)* commitment to make mechanical theory the unifying framework of physics, which means that all physical concepts, entities, hypotheses and laws of physical theories must be composed of these restricted mechanical concepts and the fundamental theoretical principles (Duhem 1892, 155); *vi)* the assumption that physics is not mathematical unless it is first mechanical. Duhem clearly states this methodological principle: "A branch of physics cannot be transformed into a mathematical theory unless it becomes a mechanical theory. For a century, this principle has guided the efforts of the physicist geometricians" (Duhem 1901, 131). Duhem disputes this principle which, for him, conflicts with the true principle that guides the construction of mathematical physics. He replaces it with what he understands to be the correct principle to make theoretical physics mathematical, the measurement theory; *vii)* an implicit commitment to physics as a rational system and method of constructing physical theory. This method, as noted above, requires the use of well-established concepts and postulates from physical theory and of rational arguments (in the form of mathematical deductions) to obtain empirical laws from its theoretical basis.

The Mechanical Theory and its Two Versions: Synthetic and Analytic

Duhem's analysis of mechanicism is organized around his criticism of two methods for applying mechanical theory to explain physical phenomena: the "synthetic method" and the "analytic method". Duhem states such an idea clearly when he says:

> The attempts made at explicating mechanically the physical phenomena that the universe presents fall into two categories. The attempts in the first category are carried out according to a method that can aptly be named the synthetic method […]; in the eyes of the majority of physicists, the synthetic method no longer seems capable of giving a mechanical and complete explication of natural phenomena; it is, then, the analytic method that one requires today for such an explication (Duhem 1980, 95; 1905, 180-181).

These are two different views of the mechanical method of constructing physical theories. Duhem's criticism of mechanicism is the criticism of these two methods of constructing mechanical theories. Each of these mechanical methods of construction leads to different views of what mechanical explanation and theoretical physics mean and aim for. Duhem critically examines the problems and difficulties affecting both approaches, evaluating them critically regarding their capacities to provide a general and unifying mechanical explanation of physics and to make the development of physics continuous and rational. Further, they are evaluated on the basis of his own view of physical theory as a rational system, that is to say a system of physical laws represented by a few formal principles.

The Synthetic Method and its Two Versions: The English School and the Classical Rationalism / Continental School

Duhem argues, from the beginning, that the use of mechanical theories is not sufficient to distinguish between the English School and the Continental School or the metaphysical view of physics. He writes: "This predilection for explanatory and mechanical theories is, of course, not a sufficient basis to distinguish English doctrines from the scientific traditions thriving in other countries" (Duhem 1974, 72; Chiappin 1989, 38-58). Both schools follow the synthetic approach to the mechanistic program. However, they differ in their ways of interpreting what theoretical physics and mechanical explanation are, how to represent their conceptual bases, and how to connect the conceptual and the empirical bases. The sharp distinctions between these schools can also be traced to distinctions between the metaphysical/rationalist and the pragmatist/empiricist view of the mechanistic program. These two views of scientific knowledge strongly influenced the various ways of interpreting the synthetic approach to mechanical theory. Undoubtedly, both views seek to explain physical phenomena according to the synthetic method. Both seek to elaborate definite, specific and detailed models of matter and motion to explain physical phenomena mechanically, the characteristic feature of the synthetic method.

However, one can reconstruct the common core of these two versions of the synthetic method to explicate physical phenomena mechanically as described by Duhem:

> In this method one begins by constructing a mechanism from all pieces; one says what are the bodies that compose it, what are their shapes, sizes and masses, and what forces act upon it, and from these data one draws the laws according to which the mechanism moves; then, by comparing these laws with the experimental laws it seeks to explain, one judges whether there is sufficient agreement between them (Duhem 1980, 95).

At the root of the synthetic method of mechanical theory to explain physical phenomena mechanically is an attempt to define specific and detailed mechanisms, or mechanical models based on definite hypotheses about the shape of atoms and molecules, their size and their arrangement. For each category of physical phenomena the synthetic method defines a mechanical explanation based upon a specific number of bodies with a specific arrangement of shapes and definite motions and mass.

These arrangements are supposed to express the causal explanation of, or to imitate and simulate, physical phenomena, or, "in the words of English physicists, [to be] a model" (Duhem 1980, 102). These mechanical models or mechanisms may further be made of dimensional and real elements, such as fluids or corpuscles with definite sizes, shapes and masses. When these models are made of plastic or wood, or drawn, they are called scale models. The use of real and concrete models is a trademark of the English School. Although this synthetic approach is not the only means of applying mechanical theory, it prevailed over every other and is practiced by most of those working on mechanistic research programs. According to Duhem, Poincaré was one of its representatives and introduced the English School (pragmatism) in France (Duhem 1974, 319, 328; Chiappin 1989, 130-134; 136-150).

As a result, most mechanical explanations are based on this method of constructing physical theory. The examples abound. Duhem cites Descartes's theory of magnetic attractions and repulsions, Descartes's explanation of weight by vortex action, and Kelvin's gyrostatic ether (Duhem 1980, 96). Other instances are Maxwell's thermodynamical surfaces used to describe Gibbs's phase rules, Descartes's mechanical explanation of the properties of light, Laplace's physical theory, and, most outstanding, Maxwell's cellular constructions, with which he attempts to account for electromagnetic actions. Maxwell's description can be found in his memoir entitled "On Physical Lines of Force" (Maxwell 1952). This model simulates a mechanism put forward to explain (mechanically) electrostatic and electromagnetic effects.

An understanding of the differences between these two views is central to understanding Duhem's view of a conception of physical theory as an intermediary or a middle way (*tertium*) between them.

The Continental School

Besides the characteristic features of the synthetic method, i.e. the use of a restricted number of mechanical concepts applied through the definite properties of a specific material system, the Continental School (Duhem 1893, 352; Chiappin 1989, 40) requires that mechanical explanations "be subject to certain logical requirements" (Duhem 1974, 78; 1893, 358). For example, all mechanical concepts and empirical laws must be organized into a single, rigid axiomatic system, made of well-established concepts and principles, and all empirical laws must be mathematically deduced within this axiomatic system. The Continental School follows the rationalist tradition as to the nature of physical theory, meaning that physical theory is modeled on the ideal of the rational system of Euclidean geometry. Physical theory is a rigorous axiomatic system that logically unites its definitions and postulates with their testable consequences. Duhem says, "For a Frenchman or a German, a physical theory is essentially a logical system. Perfectly rigorous deductions unite the hypotheses at the base of a theory to the consequences which are derivable from it and are to be compared with experimental laws" (Duhem 1974, 78).

The Continental School requires the basic concepts of physical theory to be quantitative in order that algebraic language may be used. Basic concepts are mechanical concepts because they are quantitative. Mechanical theory is thus both a rational and algebraic system. The Continental School also follows the metaphysical view associated with this rationalist tradition, which defines the nature of physical theory as aiming at the causal explanation of physical phenomena. Duhem describes how "The French geometers who composed the first theories of mathematical physics had a tendency to see theories as true explanations in the metaphysical sense. They assumed that they had reached the reality of things and the true causes of phenomena. This tendency begun by Descartes was evident in the work of Laplace and Poisson, of Fresnel, Gauchy and Ampère" (Duhem 1893, 358).

The metaphysical commitment of the Continental School to the principle "of the identity of the real and the intelligible" (Duhem 1974, 320) requires that basic mechanical concepts represent the real causes in nature. The content of physical theory is given by assuming that the primary concepts (mass, shape, motion and/or force) and principles which are composed of these concepts represent real mechanical properties of a material system and the laws of nature governing masses in motion. They assume that there are causal principles for all physical phenomena. All remaining physical properties must be reduced to these basic mechanical concepts. According to this view, physical theories are explanatory (Duhem 1974, 80).

As mentioned, it was Descartes who developed the ontology and epistemology to legitimate this view and define methodologically the characteristic features of its physical theory and method of construction. Both demands, rationalist and metaphysical, establish fundamental differences between the Continental School's view of physical theory and the English School's view. The rationalist tradition of the Continental School constructs physical theory according to rules of geometrically inspired principles such as abstraction, simplicity, coherence, and logical unity. These principles are also metaphysical principles, in that they are assumed to be properties in nature (as described by Descartes) (Duhem 1893, 352).

Among the rationalist virtues with which the Continental School expects physical theory to comply, the logical unity of physical theory is foremost. This principle rejects any contradiction and incompatibility within as well as between theoretical systems. It requires that the laws be not contradictory, that they be independent and mutually consistent. This principle pervades all the different physical theories of the Continental School.

Mechanical theory, for the Continental School, is the unifying framework of physics, and most of its members make the mechanical theory of matter the means by which the mechanistic program seeks to unify the whole of physics. From Descartes to Poisson, the principle of logical unity takes different forms, depending on the details of the theory of matter, but it stands as an unquestionable category to shape physical theory. The principle of unification in Descartes's view is matter and motion and its set of fundamental mechanical principles. They are the basis for causal explanation for every physical phenomenon: gravity, light, magnetic attractions and repulsions (Duhem 1980, 472-473).

Laplace grounds the principle of unity in his mechanical-molecular model of matter with central forces and Newton's vectorial mechanics. Lagrange differs from Laplace in that he rests the principle of logical unity not precisely on a theory of matter but rather on a system of generalized coordinates, generalized forces, the principle of virtual velocities (Duhem 1905, 42) and D'Alembert's principle (Duhem 1905, 62). Duhem rejects the unification of physical theory by a theory of matter. He is committed to the goal of unification by means of mathematical properties: stability and equilibrium imposed by thermodynamics. Duhem makes use of Lagrange's method, which is called energetic mechanics – his method to construct the energy of the system –, and makes the potential of the system its main concept.

A theoretical physics developed according to Lagrange's method of potential and without a theory of matter is present in Gibbs's formulation of thermodynamics. The Gibbsian view of thermodynamics is the model of unification according to Duhem. For the Continental School, the principle of logical unity is the leading principle for the construction of physical theory. In accordance with this principle, "for a mathematician of the school of Laplace or Ampère, it would be absurd to give two distinct theoretical explanations for the same law, and to maintain that these two explanations are equally valid" (Duhem 1974, 81).

The principle of logical unity is understood by the rationalist tradition as a logical principle whose violation is against the laws of reason and is therefore absurd. This principle demands that incompatibilities between theories/laws and concepts be eradicated from physical theory, and that all physical phenomena be deduced from the concepts and fundamental principles of mechanics.

The English School

The nature and character of the English School's mechanical explanation of physical phenomena (Chiappin 1989,43) is spelled out by Lord Kelvin when he says that

> It seems to me that the true meaning of the question "What do we understand or not understand by a particular subject in physics?" is this: Can we make a corresponding mechanical model? […] I am never satisfied as long as I have been unable to make a mechanical model for the object; if I can make a mechanical model, I understand; for as long as I cannot make a mechanical model, I do not understand (in Duhem 1980, 102).

Therefore, model-building is the method of construction of physical theories, and physical theory is identified with the constructions of models. The synthetic application of mechanical theory in the English School's view is influenced by the empiricist tradition stemming from Bacon, Locke, Hume and Newton. Particularly, the experimentalist conception of physical theory is taken from Newton's optics. At the root of this experimental/empirical conception of physics is the methodological principle of the separability and testability of isolated hypotheses, of crucial experiments, and of the inductive method. This principle can also be associated with Newton's basic view of mechanics as applying to isolated particles. The fundamental laws of mechanics as stated by Newton hold in the first instance for a single particle only (Lanczos 1986, 4).

The English School took from the empiricist tradition the determination to emphasize the empirical/practical aspects of physics over the theoretical. Moreover, for the English School, empirical adequacy should be the relevant criterion to accept mechanical explanations over rationalist criteria. Most of the English scientific community in the 19th century did not take formal and rationalist values as meaningful criteria to construct physical theory. Logical unity and other rationalist virtues such as axiomatization, consistency and simplicity were considered of no great importance to elaborate a mechanical explanation of physical phenomena.

For this school, as opposed to the Continental School, a mechanical explanation is not given by a well-constructed system of propositions logically chained from its definitions and postulates to its testable consequences, but rather by means of a sequence of disparate, concrete and figurative models. The English School does not require rational agreement between the mechanical system and the empirical laws which it is supposed to explain. Physical theory is identified with mechanical models made of concrete and real elements. The English School defends the methodological right to construct for each category of physical phenomena one or more mechanical models instead of a single mechanical model. Underdetermination of models by data is a methodological resource to construct physical theories. Maxwell and Lord Kelvin are outstanding representatives of this school, and their physical theories are a sequence of disparate models. Duhem states: "for a physicist of the school of Thomson or Maxwell, there is no contradiction in the fact that the same law can be represented by two different models" (Duhem 1974, 81; 1893, 81, 361).

In other words, they replace the principle of logical unity with the principle of the underdetermination of theory by data. Incompatibilities and contradictions between models are not violations of any logical principles, but instead they are methodological resources for the construction of mechanical explanations. The English School also does not demand rational agreement between the conceptual basis of mechanical theory and its empirical basis of empirical laws. This connection is made by mechanical models that simulate or show resemblances to these physical laws.

The relevant criterion to accept mechanical models as a mechanical explanation is that of "resemblance" between the model and the physical phenomena thereby represented. Empirical adequacy is interpreted as a pictorial way of simulating physical phenomena by models rather than a logical agreement between experimental laws and mechanical principles. The model-building method to construct physical theories is not concerned with logically rigorous hypothetical-deductive derivation of the empirical laws from the conceptual system of mechanics (Duhem 1893, 353-354).

Thus, the fundamental differences between the rationalist-oriented Continental School and the pragmatist/empiricist-oriented English School can be summed up as follows: the definition of the nature of physical theory; the use of the principle of logical unity in physical theory; the method of the construction of physical theory; and the cognitive value of mechanical theory.

The Continental School follows the rational method while the English School uses the piecemeal method of model building. While for the Continental School "it would be absurd to give two distinct explanations for the same law and to maintain that these two explanations can be true at the same time, for an English physicist there would be no contradiction in the fact that the same law can be figured out in two different manners by two different models" (Duhem 1893, 360).

The Continental School applies the principle of logical unity while the English School applies the principle of the underdetermination of theory by data. In the wake of its rejection of the principle of logical unity, the English School demolishes another much-loved rationalist value, namely the notion of physical theory as a rational system. Duhem says: "Theory is, for [the English School], neither an explanation nor a rational classification of physical laws, but a model of these laws, a model built not to satisfy reason, but to please the imagination" (Duhem 1974, 81;1893, 361).

Duhem stresses the fact that the model-building method is a non-rational method to construct physical theory, and that physical theories constructed in this manner are non-rational systems, unable to provide a rational classification of empirical laws. They must, therefore, be rejected. This is one of his harsh criticisms of the English School. "From there, these discrepancies, these incoherencies, these contradictions generated by English theories, that we tend to judge severely because we are looking for a rational system when the author only intends to give us a work of imagination" (Duhem 1893, 361).

Duhem's defense of the rational method of construction of physical theory is essential because it reflects the true nature of his conception; his rationalist commitment gives rise to a demarcation from the conventionalist/instrumentalist interpretation of physical theory. His rationalist commitment is what explains his commitment to applying the principle of logical unity in physical theory. Further, it is his view of physical theory as a rational system that explains his criticism of the English School for manipulating theoretical systems as algebraic models. Conventionalism and instrumentalism are committed to the principle of the underdetermination of physical theory by data.

The Continental School follows a metaphysical view of the nature of physical theory regarding its cognitive status, meaning that physical theory aims at explicating the real causes of physical phenomena. Its main representatives, Descartes, Laplace and Poisson, believe that mathematical physics provides real and causal mechanical explanations. Two concepts of truth underlie this view: the theory of truth as correspondence, which is applied to the conceptual basis of mechanical systems, and the theory of truth as coherence, which guarantees that mathematical principles and logical principles lead from truth to truth. The theory of correspondence is the rationalist principle of "identity of the real and the intelligible" (Duhem 1974, 320; 1893, 358).

The English School rejects the application of both concepts of truth because they are incompatible with the underdetermination of models. The underdetermination of models does not seem methodologically compatible with the idea that the mechanical elements in the models can represent the mechanical cause of physical phenomena. The principle of underdetermination is inherently an anti-realist methodological rule. The model-building method methodologically expresses its purposes: physical theories are to be considered solely as convenient instruments for experimental research. According to Duhem, the English School is concerned only with the utilitarian value of physical theory rather than with theoretical knowledge (Duhem 1974, 319).

The English School does not regard mechanical models as solutions for the problem of the identity of the real and the intelligible. According to Duhem, mechanical models are used as solutions to the problem of providing convenient instruments for experimental research, and are means to act on nature rather than to know nature. The underdetermination principle is the primary anti-realist argument. Consistent with its methodology, the English School assumes a pragmatist/empiricist view of the value of physical theory in which models have the status of a recipe, a practical and instrumental value. The pragmatist/empiricist view of the cognitive value of physical theory is reinforced by the fact that the metaphysical/aprioristic approach to the theory of matter which underlies mechanical theory suffered various theoretical as well as experimental setbacks.

The English School also questions the idea that the experimental method is a means to decide the truth value of scientific propositions. It is a method to construct empirical laws and models resembling these laws. These models and laws are interpreted as guides to act on nature rather than means to know nature (in its metaphysical sense). According to Duhem, Poincaré not only introduces the English School view in France, but further develops its epistemological and methodological assumptions (Duhem 1974, 86-93, 149, 251, 319; Chiappin 1989, 320, 130-134;). Duhem's goal is to avoid this conception of physical theory (Duhem 1974, 149; Chiappin 1989, 320). He must pursue this goal without committing himself to the metaphysical aspect of the rationalist view of physical theory. His proposal pursues a middle way between the metaphysical view and the English (pragmatist/conventionalist) views, retaining from the pragmatist/empiricist view its criticism of the causal explanation of physical phenomena.

The problem for him is how to construct a view of the nature of physical theory which methodologically rejects the search for causes without becoming, at the same time, a pragmatical view. The English School's assumption of the principle of underdetermination is a natural methodological resource against such an explanatory view of physical theory. Duhem's problem is to find a similar methodological resource which naturally rejects the causal view of physical theory (its realist component, the entities) without giving up the application of the principle of logical unity in physical theory, for logical unity is the core of Duhem's view of physical theory as a rational system and of his view of scientific growth as rational and continuous. The principle of logical unity presses the search for more and more comprehensive physical theories.

Therefore, despite the epistemological and methodological differences between these two views of the synthetic method of constructing physical theory – one associated with the metaphysical/rationalist and the other with the pragmatic/empiricist view –, Duhem shows that neither can comply with the principle of the continuity of scientific development, nor with the aim of a unifying framework for the whole of physics (Duhem 1917, 152). In particular, he shows that the model-building method is not a rational method of constructing physical theory, and, accordingly, that any physical theory constructed by this method is not rational.

One can sum up his criticisms of the synthetic method by saying that this method does not make scientific progress continuous. A physicist, says Duhem, "Will see [the physical theories constructed by the mechanistic method] constantly being reborn, but constantly aborted; […] it will clearly appear to him that the physics of atomism, condemned to perpetual fresh starts, does not tend through continued progress to the ideal form of physical theory" (Duhem 1974, 304). Duhem also argues that it cannot comply with the principle of the logical unity of physics. He says that the attempt of the synthetic method to provide a unification of all empirical laws makes its explanation overburdened with arbitrary and bizarre combinations (Duhem 1974, 304).

The Analytic Method: Description and Problems

According to Duhem, the second attempt at explicating physical phenomena mechanically is carried out by the analytic approach (Duhem 1980, 96; Chiappin 1989, 57-80). This new attempt arose in the English School when the aforementioned difficulties began to emerge from the synthetic approach to mechanical theory. Maxwell himself was disappointed with the results of his attempt at a mechanical explanation according to the synthetic approach. Duhem says: "Undoubtedly, therefore, Maxwell found little satisfaction in the mechanism he had thought of, for he soon abandoned it to set out upon a completely different path toward the mechanical explication of electric phenomena" (Duhem 1980, 68-69). Thus Duhem and even Poincaré point out that the complicated and bizarre forms taken by the mechanical models are some of Maxwell's reasons for such a disappointment. Poincaré says: "The strangeness and complication of the hypotheses that he had been compelled to make had led him to give them up" (Poincaré 1901, ix).

Certainly Poincaré does not care about the problems of the synthetic method in regard to the principle of logical unity and the principle of historical continuity because he is not methodologically committed to these principles. Maxwell seeks to develop, in order to avoid the difficulties associated with the use of mechanical models, a mechanistic method that will provide a more abstract approach to constructing physical theory. By this time mechanics already had an abstract method to construct mechanical theories, namely the Lagrangian/analytic mechanics. Maxwell adopts analytic mechanics to explain physical phenomena, giving rise to a new view of mechanical explanations. The application of analytic mechanics to the construction of mechanical theories constitutes a new version of the mechanistic program. The core of these new mechanical explanations consist of mechanisms of masses and motions.

The epistemological and methodological consequences of this analytic method are far-reaching. One of them is that the arbitrariness and indeterminacy of these invisible mechanisms of hidden masses and motions (Maxwell 1867, 50) make any empirical law susceptible to a mechanical explanation. In our view Poincaré develops the appropriate epistemological and methodological details of this view of the nature of physical theory: a pragmatist/conventionalist view.

Duhem proposes to disenfranchise Poincaré's generalization of the consequences of his critical analysis of this method applied in mechanical theory to physics in general. This generalization would be legitimate if mechanical theory were the unifying framework for the whole of physics, or if it were based on purely mathematical terms. Duhem questions Poincaré's view because it is based upon strategies appropriate to mechanical theories and not to theoretical physics such as mathematical physics.

Maxwell's methodological viewpoint on the mechanistic program (like many of his contemporaries') uncritically presupposes a generalized thesis about the relation between Lagrange's formulation of mechanics and the mechanical interpretation of physical phenomena. This thesis identifies the Lagrangian method with the mechanical interpretation of physical phenomena, constituting the main methodological principle of the analytical approach to mechanical explanation.

Duhem begins his critical examination of the analytic approach to mechanical theory by questioning this identification, and in particular its sufficient condition. Duhem separates the mathematical structure of Lagrange's method from its mechanical interpretation (Chiappin 1989, 62-68). The mechanical interpretation is, for him, one of the many possible interpretations of the Lagrangian system of equations and generalized coordinates. His criticism illuminates the generality of the mathematical formulation of Lagrangian mechanics by pointing out that its applications go beyond the application in mechanical theory (for example, Maxwell's use of it to organize his electromagnetism theory). The axiomatization of thermodynamics by Gibbs was based upon thermodynamical potential instead of Carnot's cycle. The model of this axiomatization of thermodynamics is the Lagrangian method and constitutes Duhem's ideal of theoretical physics. The Lagrangian method constitutes Duhem's ideal of mathematical structure for the organization of empirical laws.

Duhem is also particularly concerned with the unfalsifiable character of physical theories produced by the application of the analytical method, and with its strategy to avoid the falsifiability of physical theories (Duhem 1980, 78-79; Chiappin 1989, 74-80). This comes from the identification of Lagrange's method of mechanics (Lagrange 1997) with the theory of matter (Chiappin 1989, 64). Unfalsifiable theories do not meet Duhem's view of scientific growth as rational and continuous.

The key element of the analytical approach to mechanical explanation is analytic mechanics according to Lagrange's and Hertz's formulations. The Lagrangian formulation of mechanics corresponds to the foundations of mechanics according to the dynamical current of mechanics (Newton), while Hertz's formulation corresponds to the foundations of mechanics according to the kinetic current of mechanics (Descartes). Whatever its formulation, the analytic approach to the mechanistic program aims at establishing the possible conditions to explain a physical phenomenon mechanically, rather than effectively construct a mechanical explanation, as with the synthetic approach. This new approach seeks, instead of building up a mechanical model (like that of honeycomb or idle wheel particles proposed by Maxwell to explain currents and fields) (Duhem 1980, 68), to construct an algebraic equation for the kinetic energy (T) and potential energy (U) of the physical system. The Lagrangian formula is defined as $L = T - U$. Lagrange's equation is given by $\left(\frac{dL}{dq}\right) - \frac{d}{dt}\left(\frac{dL}{dq'}\right) = 0$, where $\left(q, q^{'}\right)$ describes its generalized coordinates. From this equation, one can mathematically derive through Lagrange's equations the empirical laws of physical phenomena discovered by the experimental method, and then search for mechanical interpretations or analogies.

The mathematical equations and the transformation laws involved in the analytical approach to mechanics enable a strategy to defend a unified mechanical view of physics. Maxwell himself explicitly makes this commitment to the application of the analytic approach to mechanics in his *Treatise on Electricity and Magnetism* (Maxwell 1954). There are now two new logical goals brought by the application of analytical mechanics to electromagnetism. First, to apply the analytic approach, which is concerned with measurements and mathematical relations concerning physical quantities; and second, to be committed to the establishment of the connections and analogies between this analytic approach and the mechanical explanation of physical phenomena.

Poincaré synthesizes, in his analysis of Maxwell's electromagnetism, these conditions according to the analytical method to make physical phenomena susceptible to mechanical explanation. This is precisely a definition of the analytic approach to mechanicism:

> It is easy to understand now what Maxwell's fundamental idea is. To demonstrate the possibility of a mechanical explanation of electricity, we did not need to worry about finding the explanation itself, it was enough to know the expression of the two (T) and (U) functions, which are the two parts of the energy, to form with these two functions Lagrange's equations, and then compare these equations with the experimental laws (Poincaré 1901, viii).

There is a meaningful assumption underlying this entire discussion, namely the thesis of the identification between Lagrangian representation and mechanical theory. This thesis is supposed to guarantee the existence of a mechanical explanation as soon as one constructs the Lagrangian function of the system. Thanks to this thesis, Poincaré shows that the analytic approach to the mechanistic program brought into evidence this methodological truth: "if a phenomena has a complete mechanical explanation, it will have an infinity of others which give an equally good account for all the particularities revealed by experiment" (Poincaré 1901, viii). This methodological conclusion contains the core of Poincaré's pragmatist/conventionalist view, the principle of the underdetermination of physical theory by data (Pareto 1909, 31-36), which Duhem's theory of science strives to make illegitimate.

So what one must understand by a mechanical explanation of physical phenomena is the possibility of constructing an independent system of equations made up of kinetic (T) energy and potential (U) energy and certain numbers of parameters (generalized coordinates). Assuming that one can construct such equations, one can always, according to Poincaré's analysis of Maxwell's analytic method, determine masses (hidden or visible) and their generalized coordinates in such a way that the kinetic and potential energy of this system of masses is equal to that of the kinetic (T) and potential (U) energy appearing in Lagrange's equations (Poincaré 1901, viii).

From these conditions and from Lagrange's equation one draws the equations for the motion of the system. If these equations are identical with the empirical laws constructed by the experimental method, then, according to Maxwell, "we shall have proved that electromagnetic phenomena are capable of a mechanical explication" (Duhem 1980, 70). Duhem questions that these conditions prove that a physical phenomenon is susceptible to a mechanical explication. All one has here is a mechanical interpretation or analogy.

Duhem's Analysis and Objections to the Analytic Approach to Mechanicism

Duhem begins his criticism of the analytic approach by questioning the identity between analytic mechanics and mechanical explanation. First, though, it should be recalled that, with this approach, attempts are made to avoid the aforementioned difficulties arising from the synthetic approach. Those difficulties associated with the complicated and bizarre form of mechanical models (Duhem 1980, 68-69; Poincaré 1901, ix; Chiappin 57-58) arise from the demand of making conjectures about, or simulating, the particular mechanisms underlying the physical phenomena being studied. Nothing similar is done in the analytic approach: there is no demand to construct a mechanical model. The task is to construct the algebraic equations of the kinetic and potential energy of the system and to apply them to Lagrange's equations.

The construction of these equations employs the measurable quantities of observable physical phenomena. The agreement between the empirical laws of physical phenomena discovered by the experimental method and the equations deduced (through Lagrange's equations) from the kinetic energy, potential energy, and virtual work constructed from these same measurable quantities is then assumed to guarantee a mechanical explanation for the phenomena. Since these conditions are fulfilled, one can construct, in principle, a mechanical system of masses in motion with the same kinetic energy and potential energy of the physical phenomena studied. Therefore, according to Maxwell, a mechanical correspondence between the two sets of quantities involved is warranted, which is Maxwell's concept of the reduction of physical theories. This reduction is guaranteed by three assumptions: the thesis of equivalence, the construction of Lagrange's equations, and the agreement between the constructed laws and the empirical laws.

According to Duhem, the significant mistake (confusion) here is the identification between the Lagrangian method and mechanical interpretation (Chiappin 1989, 61-79). Underlying this criticism, one finds his view that theoretical physics is a mathematical structure with a physical interpretation. He understands Lagrangian mechanics as an interpreted calculus (variational calculus), whose mathematical structure can be applied to different domains of physics.

Duhem questions the thesis of the equivalence between analytic mechanics and mechanical interpretation and its condition of sufficiency. The first major objection to the analytic approach to the mechanistic program is that the formulation of a physical problem in Lagrangian mechanics is not a sufficient condition to guarantee its mechanical explanation (Duhem 1980, 71).

Duhem agrees that it is a necessary condition, but disagrees that it is a sufficient condition. He questions the thesis that from this condition one can conclude with certainty that there exists a certain group of masses and forces, a certain mechanism, admitting such potential, and, above all, such kinetic energy (Duhem 1980, 7).

The proposed mechanical explanations were to be interpreted as illustrations, which, at most, imitate or simulate in their laws of motion the equations that are being discussed, and are not true mechanical explanations. Duhem says:

> It seems imprudent to dismiss similar difficulties with a stroke of the pen. What has been found to be best, up to now, for clearing objections of this nature, is to imagine extremely simple mechanisms whose internal potential and kinetic energy offer, in their various particularities, a more or less direct analogy with the potential and kinetic energy that it is proposed to study; in a word, this is to construct models which imitate in their laws of motion the equations that are discussed. Aided by the theory of monocyclic systems, Boltzmann has illustrated the views of Maxwell on the analogy between Lagrange's equations and the laws of electrodynamics within such models (Duhem 1980, 72).

Duhem draws attention to the difference between a truly mechanical explanation, which would be the real causal explanation of the physical phenomena, and mechanical illustrations/models, which are mechanisms with the similar potential and kinetic energy of the system studied. Further, we would like to point out that Duhem's view of the generality and extendibility of the Lagrangian method underlies his criticism of the thesis of equivalence.

Duhem considers that the Lagrangian method is a mathematical structure that can be applied to different domains of physical quantities and kinds of forces where the generalized system of coordinates receives different physical interpretations. Mechanical interpretation would be only one particular interpretation.

Duhem himself applies Lagrangian formalism to make thermodynamics an axiomatic system, without mechanically interpreting the phenomenon of heat. The conditions of the extendibility of the Lagrangian method is the subject of his article "Sur quelques extensions récentes de la statique et de la dynamique" (Duhem 1901, 130-157) and is also his preoccupation in the second part of his book *The Evolution of Mechanics* (Duhem 1980, 105-189; 1905). His conception of physical theory arises from his understanding of this generality of Lagrangian mechanics. His view of the scientific development as pursuing more comprehensive abstract theories is based upon the transition from Newton's vectorial mechanics to Lagrangian mechanics.

This objection is not, however, the only objection that he raises against the mechanistic interpretation of the analytic approach. Duhem points out a methodological problem in this method of constructing physical theory. The analytic method constructs unfalsifiable mechanistic theories, and leads the mechanistic method to a process of infinite regression. The analytic approach to the mechanistic method, which aims to make mechanical theory a unifying principle of physics, gives rise to the following question "can all physical laws be put into the form of Lagrange's equations?" (Duhem 1980, 73). In other words, can the analytic approach to the mechanistic program provide the means to accomplish the aim of unifying the whole of physics? There are two answers to the above question.

The first is pursued by some physicists, like Poincaré, who state that "There exists a radical incompatibility between Lagrange's mechanics and the laws of physics" (Duhem 1980, 73; Poincaré 1892, xviii). These physicists point out the difficulty in reducing the second law of thermodynamics (and all irreversible phenomena) to mechanics as evidence of this fact. They reason that if, on the one hand, an analysis of Lagrange's mechanics shows that "all motions controlled by the dynamics of D'Alembert and Lagrange are reversible motions" and if, on the other hand, the examples given by the experimental method and the facts show that "natural motions are not reversible" (Duhem 1980, 75), then there is an evident incompatibility between the facts and the mechanical theory. What follows from this incompatibility? Is it that one should stick to mechanical theory as the unifying framework of physics and attempt to modify its basic principles, such as D'Alembert's principle (Duhem 1980, 75-76)? Let us not go too fast here.

According to Duhem, a more detailed logical analysis of the analytic approach to mechanical theory, for example the one carried out by Helmholtz's analysis of the mechanical program, shows that there is no possible justification for such an incompatibility. Based upon Helmholtz's analysis, Duhem opposes the thesis of the incompatibility between the analytic approach to the mechanistic method of constructing physical theories and natural motions. According to Duhem, to show that such incompatibility is only apparent from the viewpoint of the foundations of analytic mechanics is enough to admit the hypothesis of the atomistic nature of matter (Duhem 1980, 78).

As mentioned the arbitrariness and indeterminacy of these invisible mechanisms of hidden masses and motions (Maxwell 1867, 50) make possible to construct a mechanical explanation for any empirical law. Duhem says: "Whatever may be the form of the mathematical laws to which experimental inference subjects physical phenomena, it is always permissible to pretend that these phenomena are the effects of motions, perceptible or hidden, subject to the dynamics of Lagrange" (Duhem 1980, 78). The indeterminacy and arbitrariness of the invisible mechanism of hidden masses in motion make theoretically possible the finding of mechanical explanations for any possible discrepancies with empirical facts (Duhem 1980, 77-78).

Is this appeal to invisible mechanism repugnant to a physicist? Not at all. It is exactly what physicists have been doing, and examples in physics are abundant. This is the case in the work of Helmholtz, Boltzmann, Clausius, and Maxwell (Laranjeiras 2002; Laranjeiras and Chiappin 2006). Helmholtz employed the mechanistic program according to the analytic method, where the hypothesis of a system of hidden masses in motion assumes the form of monocyclic systems, generated by Lagrange's equations, to build up mechanical illustrations, for instance of the second law of thermodynamics (Laranjeiras and Chiappin 2008). Helmholtz's work was the source for Boltzmann, Clausius, and Gibbs. Therefore, for Duhem, physicists such as Poincaré methodologically interpret incorrectly the incompatibility between mechanism and facts. There is no possible incompatibility between mechanism and experimental facts. The mechanistic method based on Lagrange's mechanics and on the indeterminacy and arbitrariness of the system of hidden masses in motion is not falsifiable in any way (Duhem 1905, 182). This, for Duhem, is the correct interpretation of the analytic approach to the mechanistic method. Thus, he points out that the consequence of using this hypothesis will be the unfalsifiable character of mechanical theories constructed by the analytic approach. He seeks then to preserve the mathematical formulation of D'Alembert's principle. He argues that, from the algebraic viewpoint, modifications and generalizations of the equations of dynamics by introducing the term of first degree in the velocities are easy to notice (Duhem 1980, 76).

Thus, Duhem preserves the basic principles of Lagrangian mechanics and redirects the difficulties and obstacles raised against the mechanistic program to its two presuppositions: the atomistic hypothesis of matter and its ultimate demand that physical concepts be reduced to a restricted number of primitive mechanical magnitudes such as mass, motion, shape and/or force (Duhem 1903, 270).

The goal of unifying the whole of physics (the principle of logical unity) by the analytic approach to the mechanistic program, with its restricted mechanical conceptual basis (mass, motion, forces), is attained at the cost of a large indeterminacy of its parameters of masses and motions. And the consequence of the large indeterminacy and arbitrariness of this system of hidden masses in motion is the unfalsifiability of mechanical theories. In the analytic approach, mass and motion are mere parameters in mathematical equations, without the realistic or figurative connotations they have in the synthetic approach. The synthetic approach to mechanical explanation involves specifications and determinations of the masses, motions, sizes and forces.

Why does Duhem see the unfalsifiability of the mechanistic method as a problem and why is it a source for his criticism? Is not this approach a legitimate heuristic approach as defended by Poincaré? Duhem does not think that the aim of theoretical physics is to promote the discovery of new laws. Such is the task of experimental physics.

Duhem pursues a rational explanation of the evolution of theoretical physics. From this viewpoint, he evaluates the methodological characteristics of the analytic approach to mechanistic theories. He does not think that hypotheses and methods that construct unfalsifiable physical theories make scientific growth rational and continuous. For him, rationality is controllability, and continuity means that the new theory encompasses everything already accounted for. Duhem's method of constructing physical theory with his rules for the formation of physical concepts establishes that the introduction of concepts in physics must be controlled by measurable conditions, and that mechanical models are to be expelled from physical theories.

As we have seen, these systems of hidden masses in motion are totally indeterminate and arbitrary, since nothing limits the nature and number of these masses in motion; this makes mechanical theories unfalsifiable (Duhem 1980, 78-79). Consequently, physical theories constructed by a mechanistic method, according to Hertz's and Lagrange's mechanics, are not subject to the control of empirical facts. This means that the application of the empiricist principle of the testability of physical theory is made meaningless by the mechanistic method of constructing physical theory. For Duhem, a rational method of constructing physical theory (Chiappin 1989, 282-310) must optimize the empirical testability of physical theories.

Therefore, from Duhem's viewpoint, the ongoing polemic between Lagrange's formulation of classical mechanics, which takes force as a primitive concept representing a real cause, and Hertz's formulation, which rejects force as a primitive concept, does not make any difference with respect to his criticism of the mechanistic method. For him, both formulations generate unfalsifiable physical theories (Duhem 1980, 97; 1905, 183).

Hence, the analytic approach and the atomistic hypothesis can always provide compatibility between a mechanical theory and facts making it unfalsifiable (Duhem 1905, 182). This thesis, the compatibility between mechanical program and facts, means, for Duhem, that a mechanistic program cannot effectively comply with the empiricist principle of empirical testability, or with its associated methodological principle. Duhem defines this principle as follows: "In physics, one criterion alone allows the rejection as false of a judgment which does not imply a logical contradiction: the record of a flagrant disagreement between this judgment and the facts of experience" (Duhem 1980, 97).

The application of this principle which governs the empirical testability of physical theory in the test of mechanistic theories illuminates Duhem's conclusion that the proposition which states "all physical phenomena are explained mechanically" transcends the physical method. By physical method Duhem understands the experimental method by which one discovers empirical laws, while the mechanistic method is a method to construct physical theories which explain or represent these laws. Thus, Duhem concludes that "It is impossible for anyone who holds to the processes of the experimental method to declare as true this proposition: 'All physical phenomena are explained mechanically'. It is also impossible to declare it false. This proposition transcends the physical method" (Duhem 1980, 97-98; 1905, 183-184).

So, Duhem's analysis of the status of the mechanistic program has far-reaching methodological consequences for his conception of physical theory. One can be cited: that the decision about the mechanistic method of construction goes beyond the experimental method. Duhem says: "If, in regard to this proposition [stating the transcendent character of the mechanistic method], one wishes to depart from a state of mind where every decision remains suspended, one will have to resort to arguments unknown to experimental method" (Duhem 1980, 98).

In summary, according to Duhem, neither the metaphysical method – the foundationalist version (Duhem 1980, 98) – nor the experimental method are able to decide the truth value of mechanistic theories. Where can the answer come from? Duhem suggests: "The degree of suitability of a method in fact is essentially a matter of personal appreciation; the particular turn of each thinker, the education received, the traditions immersed in, the customs of the environment in which he lives, all influence this appreciation to a high degree; these influences vary in the extreme from one physics to another" (Duhem 1980, 99). It is difficult to avoid thinking that Duhem is the source of Popper's assertion that conceptions of science are a methodological and conventional matter. The important aspect of this discussion about the analytic method of constructing mechanical theories is Duhem's awareness of the unfalsifiable character of this method, and that the decision to reject it goes beyond the experimental method (Duhem 1974, 293-294; 1893, 366; Chiappin 1989, 134). Further, for Duhem, the analytic approach to the mechanistic method of constructing physical theory is the source of its unfalsifiability. Poincaré sticks to the mechanistic method to analyze physical theories and reinterprets the experimental method as unable to decide conclusively the truth value of scientific propositions.

Duhem defends the position that the experimental method can reject (well-constructed) physical theories, and he turns down the use of methods and hypotheses that introduce indeterminacy and arbitrariness into physical theory (Duhem 1980, 97). Duhem defines a method of constructing physical theories which makes the experimental method effective in refuting theories (Duhem 1974, 78; Chiappin 1989, 287).

Duhem does not affirm that the unfalsifiability of physical theory is a characteristic of any method of constructing physical theory. For him, the experimental method cannot reject physical theories constructed by the mechanistic method. But it is not true that the experimental method cannot aid in the rejection of any theory. The refutability of physical theories depends on their method of construction. He defends the idea that we can control the process of constructing physical theories. And from the logical viewpoint, well-constructed physical theories can be refuted (Chiappin 1989, 62-80). If theories are not falsifiable, it is our own fault. It is essential, for Duhem's view of scientific progress, that the experimental method can refute and reject physical theories. This is also essential to construct physical theories that satisfy the principle of empirical testability. This is so because Duhem is committed to the principle of continuity. From his arguments against mechanicism one can see, now, that Duhem is strongly committed to a view of scientific growth as rational and continuous. Therefore, the true methodological verdict of his historical-critical analysis of the analytic approach to the mechanistic program is that one cannot, due to its use of the atomistic hypothesis, scientifically decide whether to accept or reject it in a conclusive manner. (Chiappin 1989, 77-82)

Once it is accepted that the decision about the mechanistic program goes beyond the experimental method, one requires extra-empirical rules to legitimate this option. The notion of rational method for Duhem acquires a meaning beyond mathematical consistency and experiment. It requires extra-empirical rules.

Duhem rejects Poincaré's as well as the English School's pragmatical conception of physical theory (Duhem 1974, 149; Chiappin 1989, 51-57). He also rejects the metaphysical view of classical rationalism (Chiappin 1989, 40-42). His strategy is to reject the metaphysical view, as well as the model-building view of the nature of physical theory, and to rescue the ideal (from classical rationalism) of physical theory as a rational system based upon a very small set of formal principles in the style of the principle of least action or the potential functions of thermodynamics.

To conclude our appraisal of Duhem's position, for him the mechanistic program faces an unsolvable dilemma: if the mechanistic program wants to use the mechanical theory according to the synthetic method, then it must give up the principle of logical unity. Consequently, the rejection of the mechanistic program is, ultimately, a methodological/epistemological decision and not a scientific one.

If the mechanists want to use the mechanical theory according to the analytical method, containing the atomistic hypothesis of matter, rescuing thereby in principle its purpose of providing a unifying principle for the whole of physics, then they must give up the aim of providing a mechanical explanation proper, i.e. a picture or model of physical phenomena. Duhem states this situation clearly:

> Hence the analytic method, which alone seems capable of providing from the laws of physics a logically constructed mechanical explanation, seems incapable of satisfying the requirements of imaginative physicists, that is to say, of the very ones who required a mechanical interpretation of phenomena. If these physicists want, at any price, to picture the qualities of bodies in shapes suitable for geometric intuition, in shapes simple enough to be depicted in a table clearly understandable to the eyes and the imagination, they will have to renounce the hope of uniting all these representations into a coherent system, into a logically ordered science (Duhem 1980, 101).

Duhem's Formal Conception of Theoretical Physics and the Principle of Rational and Continuous Progress

This section will outline Duhem's view of the rational and continuous progress of physical theory.

As concluded before, for Duhem neither the metaphysical view of physics nor the model-building method can comply with the principle of continuity. Besides that, the model-building method cannot comply with the rational method, which rejects the resource to contradictory models in physics. Duhem preserves the mathematical formalism of the analytic approach, namely the Lagrangian formalism, and focuses his blame for the problems of this method on the atomistic hypothesis of matter. He points out that the atomistic hypothesis gives rise to strategies which make physical theories unfalsifiable and thereby traps them in a process of infinite regression. For him, the unfalsifiable character of physical theories and the process of infinite regression are real obstacles to seeing scientific progress as rational and continuous.

The continuous development of physical theory is shown, for Duhem, by the development of abstract theories. Abstract theories are mathematical structures (Lagrangian, Hamiltonian formalism) which form the mold to systematize and organize the empirical laws discovered by the experimental method.

Duhem clearly states his predisposition for the abstract aspect of physical theory as an element of continuity between physical theories. At the same time, he blames the use of atomistic theories of matter as unifying frameworks for the discontinuity of physical theory. He says:

> He [the physicist who is not content with knowing physics through the gossip of the moment] will see abstract theory, matured through patient labor, take possession of the new lands the experimenters have explored, organize these conquests, annex them to its old domains, and make a perfectly coordinated empire of their union. It will appear clearly to him that the physics of atomism, condemned to perpetual fresh starts, does not tend by continued progress to the ideal form of physical theory (Duhem 1974, 304).

Thus, the continuity of scientific progress is accounted for by viewing the nature of physical theory as representational structures (Chiappin 1989). The historical continuity of scientific development is shown by the increasing generalization and abstraction of these mathematical structures with which we organize our set of empirical laws. This view of the nature of physical theories operationalizes the idea of progress as the increasing comprehensiveness of physical theories, where the idea of increasing comprehensiveness accounts for the idea of continuity. The principle of continuity states that new theories contain the acquired knowledge and are systematized by the old theories.

Continuity, in Duhem's view, is identified with comprehensiveness. This view of progress as the increasing generalization and abstraction of physical theories, and therefore of increasing comprehensiveness, accounts for the continuity between Descartes, Galileo, and Newton; and between Newton and Lagrange. Further, this view accounts for the continuity between Lagrange and energetics. To make this clearer: from Descartes and Galileo to Newton we go from a set of disconnected general laws, such as inertial law, the law of fall, Kepler's laws, and the collision laws, to a more general and abstract structure forming a rigid axiomatic system of concepts and principles that encompass these laws.

This axiomatic structure is vectorial mechanics, that accounts for all these laws, which means that it provides the unification of the terrestrial and celestial laws. Vectorial mechanics is based upon the idea that bodies are composed of isolated mass points, two vectors, force and momentum, four laws, Euclidean geometry, and the parallelogram rule. From Newton to Lagrange, we move from vectorial mechanics to a mechanics based upon energy, generalized coordinates, which is applied to a system of bodies instead of isolated mass points. From the Lagrangian method to the energetic method, Duhem wants to move from local motions (velocity) to general motions (e.g. chemical reactions).

For Duhem, the continuity of scientific progress between the mechanistic and energetic methods (new mechanics) is obtained by using, at least as an analogy, the Lagrangian formalism to construct axiomatic thermodynamics. This task is undertaken by Gibbs, by applying thermodynamics as the unifying framework for physics. Thermodynamics, so constructed, provides the foundations of chemistry-physics, not mechanics. This successful unification gives rise to a promising program to implement thermodynamics as the new unifying framework.

Duhem interprets Lagrangian formalism, with its principle of D'Alembert and its principle of virtual work, as a powerful mathematical instrument to be applied in any physical domain without turning it into mechanical physics. The development toward the generalization and abstraction of the Newtonian structure of mechanics into Lagrangian mechanics is described in Duhem's book *The evolution of mechanics* (Duhem 1980, 22-46).

As a result of this generalization and abstraction, one gets rational mechanics as constructed by Lagrange and to which Bernoulli, D'Alembert and Euler contributed (Duhem 1980, 23). This mechanics reduces all laws of equilibrium and motion to a single principle (the principle of virtual velocities) and to a single method of calculation (variational calculus).

The mathematical structure furnished by Lagrangian formalism is quite simply formed of two scalar quantities: the "kinetic energy" and the "potential energy," along with the Lagrangian equations. Further, the problem-solving theory embodied by this method is, in general, more comprehensive and simpler than the theory of vectorial mechanics. The relationship between its theoretical elements, potential function and kinetic energy, and the empirical laws discovered by the experimental method seems to be more cohesive than in vectorial mechanics.

Indeed, there is almost a routine, already described by Poincaré, for solving problems in this analytic approach: the kinetic energy and potential function must be constructed in generalized coordinates, the Lagrangian function L formed from them and substituted into the Lagrangian equations in order to obtain the equations of motion. This routine furnishes Duhem with the ideal of a rational method to construct physical theory (Duhem 1892, 146; Chiappin 1989, 110)

There are two more features in this formalism that are of fundamental importance for Duhem's methodology. The first one, introduced by the Lagrangian approach to mechanics, is that it focuses on the system of particles instead of an individual particle, as in Newtonian mechanics (Lanczos 1970, 4). The second one is that it can be entirely derived from a single principle, namely the principle of least action. We have here a truly unifying principle for all sciences to which the Lagrangian formalism applies. These two features of the Lagrangian formalism fit in well with Duhem's two major methodological elements, namely the D-thesis and the principle of logical unity (Duhem 1974, 91; Chiappin 1989, 178, 240).

The next step was a new mathematical structure, whose essential scalar quantities are H, q and p, Hamiltonian mechanics. This Duhemian view of the continuous progress of theoretical physics by increasing the abstraction and generalization of mathematical structures receives substantial support from Arnold's book *Mathematical Method of Classical Mechanics* (Arnold 1980; Chiappin 1989, 86-87). Arnold describes Lagrangian and Hamiltonian mechanics with set theory. He demonstrates the generality of these two mathematical structures and the relation of inclusion between them. In the first part of his book (Arnold 1980, 1-52) he describes Newtonian mechanics as studying the motion of a system of point masses in three-dimensional Euclidean space.

In the next part (Arnold 1980, 53-159) he discusses Lagrangian mechanics, which is described as a mechanical system "given by a manifold ('configuration space') and a function on its tangent bundle (the 'Lagrangian function')" (Arnold 1980, 53). Commenting on the relation of the Lagrangian mechanical system and the Newtonian mechanical system, he says that: "A Newtonian potential system is a particular case of a Lagrangian system (the configuration space in this case is Euclidean, and the Lagrangian function is the difference between the kinetic and potential energies)" (Arnold 1980, 53).

In the following part of his book (Arnold 1980, 161-300) he discusses Hamiltonian mechanics, explaining that a "Hamiltonian mechanical system is given by an even-dimensional manifold (the 'phase space'), a symplectic structure on it (the 'Poincaré integral invariant') and a function on it (the 'Hamiltonian function'). Every one-parameter group of symplectic diffeomorphism of the phase space preserving the Hamiltonian function is associated to a first integral of the equations of motion" (Arnold 1980, 161). With respect to the relation between them, he says: "Lagrangian mechanics is contained in Hamiltonian mechanics as a special case (the phase space in this case is the cotangent bundle of the configuration space, and the Hamiltonian function is the Legendre transform of the Lagrangian function)" (Arnold 1980, 161).

If Duhem could have known Arnold's book, he would have seen it as the true expression of the continuous progress of theoretical physics. We assume that Arnold's view of the axiomatic foundations of mechanics (Chiappin 1989, 86-87), with its use of set theory, in terms of larger and more abstract mathematical structures, can be used to define Duhem's view of the progress of the order in which physical theory organizes empirical laws. However, there are increasing evidences for the role of the structures in the characterization of the physical phenomena, mainly, in statistical mechanics with the phase transition phenomena (Chiappin 2005, 11-15; Chiappin 1979, 134, 140, 169; Pettini, Franzosi and Spinelli 2000; Franzosi, Pettini and Spinelli 2014). This notion of progress corresponds to the epistemic component of Duhem's conception of physics. It has a value of knowledge.

There is another view of scientific progress in Duhem. This view is concerned with the subject-matter of physical theory and not with the mathematical structure which provides the mold for these laws. This other view defines progress as the accumulation of empirical laws with a special relation to the theory that resembles an algorithm. The development of physics can be understood as the search for a physical theory which provides a method of establishing a tight connection between the mathematical structure of the theory and the empirical laws provided by the experimental method. The method of thermodynamical potential provided by Gibbsian thermodynamics comes closer to this ideal of method, and Duhem contributed to it with the famous Duhem-Gibbs relation. This view of progress corresponds to the practical component of his conception of physics.

In summary, we have argued that a complete point of view of Duhem's continuous and rational development of physics can be condensed as his defense of a conception of physics according to which physics should be a representational system with very few formal principles coordinating the set of empirical laws (Duhem 1974, 19; 1902b, 5) which works as an intermediary or a middle way between the metaphysical and a pragmatist/conventionalist conception of theoretical physics (Chiappin 1989, iv, 92; Duhem 1917, 157). He constructs this formal conception of physics in such a way that he ends up with an idea of scientific progress in the form of a sequence of representational systems as structures of increasing comprehensiveness of empirical laws (Duhem 1974, 304, Chiappin 1989, 86-87;). This leads him to defend a convergent structural realism toward an ideal physical theory (Chiappin 1989, 198). Duhem's conception of the ideal physical theory is a natural classification of laws (Duhem 1974, 298; Chiappin 1989, 106-114). This convergent structural realism allows him to demarcate his conception, on the one hand, from the conception of metaphysical foundationalism, associated with classical rationalism (mainly Descartes), and, on the other hand, from the conception of pragmatism/conventionalism, associated with the English School (mainly Poincaré).

Conclusions

We have argued that a point of view of Duhem's continuous and rational development of physics requires a formal conception of physics that he defines as a representational system of empirical laws based upon formal principles (Duhem 1974, 304; Chiappin 1989, 260) This is a middle way between two conceptions to physics the metaphysical view and, on the other hand, the pragmatist/conventionalist view (Chiappin 1989, 243-247). He constructs this formal conception of theoretical physics in such a way that he ends up with an idea of scientific progress in the form of a sequence of representational systems as structures of increasing comprehensiveness of empirical laws, which leads him to defend a convergent structural realism (Chiappin 1989, 198) toward an ideal physical theory, a natural classification of empirical laws (Duhem 1893, 368-369; 1902a, 206, 1974, 270). The combination of a historical-critical approach to the study of physics with a formal conception allows him to develop this kind of an intermediary strategy with the construction of a dynamical theory of theoretical physics. It is this dynamical theory that allows him to demarcate his conception, on the one hand, from the metaphysical conception, associated with classical rationalism (mainly Descartes), and, on the other hand, from the conception of pragmatism/conventionalism, associated with the English School (mainly Poincaré).

References

Arnold, V.I. 1980. *Mathematical method of classical mechanics*. New York: Springer Verlag.

Chiappin, José R. N. 1989. *Duhem's theory of science: The interplay between philosophy and history of science.* 388 p. Thesis (PhD) – University of Pittsburgh.

Chiappin, José R. N. 1979. *Transição de fase no modelo de Ising com campo transverso*. 194 p. Dissertação (mestrado) - Instituto de Física - Universidade de São Paulo. http://www.teses.usp.br/teses/disponiveis/43/43133/tde-15072013-155425/pt-br.php

Chiappin, José R. N. 2005. *Métodos estocásticos aplicados à transição de fase*. 238p. Tese (PhD) - Instituto de Física - Universidade de São Paulo, 2005. http://www.teses.usp.br/teses/disponiveis/43/43134/tde-08122009-161152/en.php

Duhem, Pierre. 1892. Quelques réflexions au sujet des théories physiques. *Revue des Questions Scientifiques* 31: 139-177.

Duhem, Pierre. 1893. L'école anglaise et les théories physiques, à propos d'un livre récent de W. Thomson. *Revue des Questions Scientifiques* 34: 345-378.

Duhem, Pierre. 1901. Sur quelques extensions récentes de la statique et de la dynamique. *Revue des Questions Scientifiques* 50: 130-157.

Duhem, Pierre. 1902a. *Le mixte et la combinaison chimique: essai sur l'évolution d'une idée.* Paris: C. Naud.

Duhem, Pierre. 1902b. *Les Théories électriques de J. Clerk Maxwell. Étude historique et Critique.* Paris: Hermann.

Duhem, Pierre. 1903. Analyse de l'ouvrage de Ernst Mach: La mécanique, étude historique et critique de son développement. *Bulletin des Sciences Mathématiques* XXVII : 261-283.

Duhem, Pierre. 1905. *L'évolution de la mécanique.* Paris: Libraire Scientifique A. Hermann.

Duhem, Pierre. 1917. *Notice sur les titres et travaux scientifiques de Pierre Duhem.* Paris: Gauthier-Villars.

Duhem, Pierre. 1974. *The Aim and Structure of Physical Theory.* New York: Atheneum Press.

Duhem, Pierre. 1980. *The Evolution of Mechanics.* The Netherlands: Sijthoff & Noordhoff.

Franzosi, Roberto.; Pettini, Marco.; Spinelli, Leonel. 2014. *Topology and Phase Transitions I. Preliminary Results.* https://arxiv.org/pdf/math-ph/0505057v2.pdf

Lanczos, Cornelius. 1986. *The Variational Principles of Mechanics.* New York: Dover Publications.

Lagrange, Joseph Louis. 1997. *Analytical Mechanics.* Boston: Springer.

Laranjeiras, Cássio C. 2002. *O programa de pesquisa de Ludwig Boltzmann para a mecânica estatística.* Tese 267 p. de Doutorado, Departamento de Filosofia, Universidade de São Paulo.

Laranjeiras, Cássio C.; Chiappin, José R. N. 2006. A heurística de Boltzmann e a emergência do programa mecânico-estatístico. *Revista Brasileira de Ensino de Física.* 28 (3): 297-312. http://dx.doi.org/10.1590/S1806-11172006000300006.

Laranjeiras, Cássio C.; Chiappin, José R. N. 2008. A construção de uma teoria de ensembles: antecedentes em Maxwell e Boltzmann. *Revista Brasileira de Ensino de Física* 30 (1): 1601-1611. http://dx.doi.org/10.1590/S1806-11172008000100015

Maxwell, James Clerk. 1952. *On Physical Lines of Forces.* New York: Dover Publications. *The Scientific Papers of James Clerk Maxwell*. Edited by W. D. Niven.

Maxwell, James Clerk. 1954. *A Treatise on Electricity and Magnetism.* New York: Dover Publications.

Maxwell, James Clerk. 1867. On the Dynamical Theory of Gases. *Philosophical Transactions of the Royal Society of London.* 157: 49-88.

Pareto, Vilfredo. 1909. *Manuel d'économie politique.* Paris: V. Giard et E. Brière.

Pettini, Marco.; Franzosi, Roberto.; Spinelli, Lionel. 2000. Topology and phase transitions: Towards a proper mathematical definition of finite N transitions. http://www.worldscientific.com/doi/abs/10.1142/9789812810939_0024

Poincaré, Henri. 1901. *Électricité et optique: la lumière et les théories électrodynamiques. Leçons professées à la Sorbonne en 1888, 1890 et 1899.* Paris: Gauthier-Villars.

Poincaré, Henri. 1892. *Cours de physique mathématique. Thermodynamique.* Paris: Gauthier-Villars.